

Gas distribution through injection manifolds in vacuum systems

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When injecting gas into a vacuum system, quite often the gas is distributed through a gas injection manifold. However, designs normally rely upon practical experience. By considering the manifold arrangement as a network of flow restrictions it is possible to optimize the distribution of gas throughout the manifold. The methodology for determining the flow distribution through the two simplest topologies of gas manifold, single- and double-opening manifolds from a single-gas injection point, is derived in this article. It is shown that the double-opening manifold topology tends to provide more uniform flow distribution than the single-opening manifold topology for similar conductance ratios. The results of this work include a summation formula for the single-opening manifold. In addition, guidelines for one type of tailored flow manifold are given. Finally, three basic design rules are presented: (1) use as few holes in the manifold as possible, (2) use a double opening manifold when possible, and (3) specify tube dimensions such that the tube/spray hole conductance ratio is maximized. © 1995 American Vacuum Society.

I. INTRODUCTION

A common problem in vacuum reactors is the knowledge of the dispersion pattern of injected gases into the reactor through gas manifolds. Normally a manifold is utilized for injected gas components that need to be distributed uniformly along some significant axis in the reactor. One way to view a manifold is as a network of individual flow restricting elements that passively control the flow of gas. It is widely recognized that Kirchoff's laws of electrical circuits apply equally well for flow impedance in the molecular flow regime.¹⁻⁵ Some work has been performed for large components such as electron microscopes and particle accelerators, but little work has been published on gas delivery systems.³⁻⁵ This article considers two simple types of manifolds and presents the results of the distribution of the gas flow along the manifold. The first section explains how the functions describing the gas distribution are derived. It details the physical reasoning behind the use of the matrices that describe the system, as well as how the equations are applied. The second section discusses case studies for determining the "flow profile." Here the behavior of the different manifold topologies is demonstrated. In addition, a useful simplification for one spray bar topology is presented. The last section discusses how the uniformity can be controlled, either producing uniform gas injection along the length of the manifold, or the more general case of producing a tailored flow distribution.

II. DERIVATIONS

The problem of steady-state gas flow through a series of tubes and holes can be mathematically identical to resistive electrical networks. Ohm's law, $V_1 - V_2 = IR$ (where V is voltage, R is resistance, and I is current), is analogous to $P_1 - P_2 = Q\chi$ [where P is pressure, Q is mass flow rate, and χ is the inverse of flow conductance or flow impedance, (i.e., $\chi = 1/C$)].^{1-3,5,6} For any resistive network, the flow impedances add as series elements, and the conductances add for

parallel elements. Analogies of Kirchoff's laws apply, in that all flow rates going into and out of a point add to 0, and the pressure drop around a loop adds to 0. With this knowledge, it is possible to create the system of equations that describes gas distribution through any imaginable network.

Certain assumptions have been made in order to simplify calculations. The most important one is that the flow throughout the network is molecular, so conductance is heavily influenced by the component geometry. It is also assumed that conductance through each segment of the network is the same as a stand-alone component. The first assumption is used to decouple the pressure of conductance because the viscous (i.e., laminar or turbulent) gas flow regimes have pressure-dependent conductance.⁷ Since molecular flow is assumed, the mean-free-path length must be larger than the characteristic dimension for each component of the network. Finally, the system is assumed to be in steady state, so that wall loading effects can be neglected.⁸

Strictly speaking, however, the second assumption is an approximation. The resistive network analogy to calculate a lumped-sum value for conductance of several objects neglects geometric combination and molecular beaming effects. This can lead to inaccuracies of as much as 16% in the estimation of conductance.^{9,10} For example, a stand-alone component assumes that the incoming flux of gas molecules has a cosine distribution oriented on-axis with respect to the component opening, but the output beam shape is a function of component geometry that is generally not a cosine distribution. This leads to the deviation for the incoming flux beam shape from a cosine for adjacent components. Even though the purpose of this work is to understand how the conductances of various elements affect the throughput of the system, these points should be kept in mind.

The manifold simply consists of a tube or "manifold tube" that distributes gas along its length to a series of holes "distribution tubes." The two simplest types of manifold topologies are considered here: (1) a tube which has the gas injected through one end and the gas is distributed through

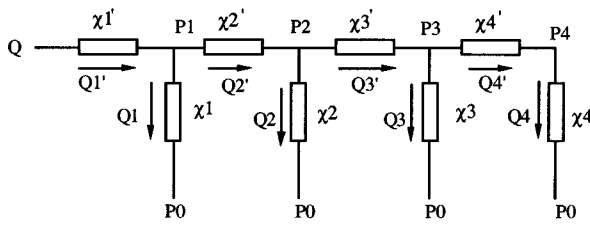


FIG. 1. Schematic of a four-hole single-opening manifold. The lines have infinite conductance; the boxes have finite conductance. The points at the bottom are the chamber pressure.

the holes in the tube (single opening), and (2) a tube with holes whose ends are both connected to the gas injection tube (double opening). When the conductances of the holes are similar to those of the distribution tubes, the distribution of gas flow through the holes is *not* close to uniform, and the distribution of gases can be affected by each additional hole.

A. Single-opening manifold

Figure 1 illustrates the case for the single-opening manifold. Gas flows from the feed tube on the left, and is distributed among all of the holes in the tube according to the flow impedances to flow of the various elements. It is assumed that the chamber pressure at each hole is the same, i.e., $\nabla P_0 = 0$. What will be determined is the dependence of the flow rate through various points of the network based upon the conductances. According to Kirchoff's current law, the flow through each node is

$$Q_1' = Q_1 + Q_2', \quad (1)$$

$$Q_2' = Q_2 + Q_3', \quad (2)$$

$$Q_3' = Q_3 + Q_4', \quad (3)$$

$$Q_4' = Q_4, \quad (4)$$

$$Q = Q_1'. \quad (5)$$

For nodes 1 to $n-1$, where i is the i th hole in the manifold, and n is the number of holes in the manifold, the node equations are $Q_i' = Q_i + Q_{i+1}'$. The pressure drops across the spray holes are

$$P_1 - P_0 = Q_1 \chi_1, \quad (6)$$

$$P_2 - P_0 = Q_2 \chi_2, \quad (7)$$

$$P_3 - P_0 = Q_3 \chi_3, \quad (8)$$

$$P_4 - P_0 = Q_4 \chi_4, \quad (9)$$

in other words, for holes 1 to n , $P_i - P_0 = Q_i \chi_i$. The pressure drop through the tube creates the following equations:

$$P_1 - P_2 = Q_2' \chi_2', \quad (10)$$

$$P_2 - P_3 = Q_3' \chi_3', \quad (11)$$

$$P_3 - P_4 = Q_4' \chi_4', \quad (12)$$

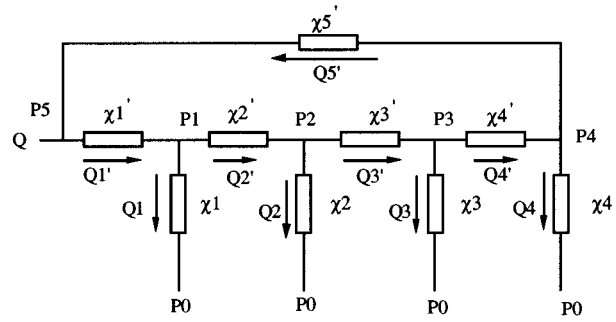


FIG. 2. Schematic of a four-hole double-opening manifold. The lines have infinite conductance; the boxes have finite conductance. The points at the bottom are the chamber pressure.

in other words, for segments 2 to n , $P_{i-1} - P_i = Q_i' \chi_i'$. These equations can be combined to form n equations to solve for the n unknowns of the flow through the manifold holes.

$$Q = Q_1 + Q_2 + Q_3 + Q_4, \quad (13)$$

$$Q_1 \chi_1 - Q_2 \chi_2 = \chi_2' (Q_2 + Q_3 + Q_4), \quad (14)$$

$$Q_2 \chi_2 - Q_3 \chi_3 = \chi_3' (Q_3 + Q_4), \quad (15)$$

$$Q_3 \chi_3 - Q_4 \chi_4 = \chi_4' (Q_4). \quad (16)$$

The system of equations translates into a matrix of Q_1, Q_2, \dots, Q_n that may be solved by Gaussian elimination.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \chi_1 & -(\chi_2 + \chi_2') & -\chi_2' & -\chi_2' \\ 0 & \chi_2 & -(\chi_3 + \chi_3') & -\chi_3' \\ 0 & 0 & \chi_3 & -(\chi_4 + \chi_4') \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (17)$$

The pivot array has 1 for the first element, and 0 for all of the other elements. Generalizing the formulation leads to the following rules. The definition of total gas flow is [from Eq. (13), where i is the hole number, and n is the total number of holes]:

$$Q = \sum_{i=1}^n Q_i. \quad (18)$$

Equations (14)–(16) are

$$Q_{i-1} \chi_{i-1} - Q_i \chi_i = \chi_i' \left(\sum_{j=i}^n Q_j \right). \quad (19)$$

B. Double-opening manifold

The analysis of an double-opening manifold network is similar to that of the single-opening tube, except that the ends are connected to form a closed loop. The connection adds an additional circuit element to the single-opening

manifold (shown in Fig. 2 as χ^5), hence an additional flow path to the network. The easiest way to solve this matrix is to build upon the previous analysis and solve for the flow through this additional element, thus requiring an additional equation. According to Kirchoff's current law, the flow through each node is

$$Q1' = Q1 + Q2', \quad (20)$$

$$Q2' = Q2 + Q3', \quad (21)$$

$$Q3' = Q3 + Q4', \quad (22)$$

$$Q4' = Q4 + Q5', \quad (23)$$

$$Q + Q5' = Q1'. \quad (24)$$

For nodes 1 to n , where i is the i th hole in the manifold, and n is the number of holes in the manifold, the node equations are $Qi' = Qi + Q(i+1)$. However, for the node at $P5$, where Q and $Q(n+1)'$ flow into the node and $Q1'$ flows out, the flow equation is $Q + Q(n+1)' = Q1'$. The pressure drops across the spray holes are the same as Eqs. (6)–(9); in other words, for holes 1 to n , $Pi - P0 = Qi\chi_i$. The pressure drop along the tube is the same as Eqs. (10)–(12) with the addition of Eqs. (25) and (26):

$$P4 - P5 = Q5'\chi_4', \quad (25)$$

$$P5 - P1 = Q1'\chi_1'; \quad (26)$$

in other words, for segments 2 to n , $P(i-1) - Pi = Qi'\chi_i'$. These equations can be combined to form n equations to solve for the n unknowns of the flow through the manifold

holes. However, it has been found to be generally easier to set up the matrix for $n+1$ unknowns by including the fraction of the gas stream that flows through tube segment ($Q5'$).

$$Q = Q1 + Q2 + Q3 + Q4, \quad (27)$$

$$Q1\chi_1 - Q2\chi_2 = \chi_2'(Q2 + Q3 + Q4 + Q5'), \quad (28)$$

$$Q2\chi_2 - Q3\chi_3 = \chi_3'(Q3 + Q4 + Q5'), \quad (29)$$

$$Q3\chi_3 - Q4\chi_4 = \chi_4'(Q4 + Q5'), \quad (30)$$

$$Q4\chi_4 - \chi_1'(Q1 + Q2 + Q3 + Q4 + Q5') - Q1\chi_1 = \chi_4'(Q5'). \quad (31)$$

Generalizing the formulation leads to the following rules: the definition of total gas flow is from Eq. (27) [the rewritten form of Eq. (18)], and the other flow equations (for holes $i=2$ to n) can be written as

$$Q(i-1)\chi(i-1) - Qi\chi_i = \chi_i' \left(Q(n+1)' + \sum_{j=1}^n Qj \right). \quad (32)$$

Equation $n+1$ is

$$Qn\chi_n - Q1\chi_1 - \chi_1' \left(Q(n+1)' + \sum_{j=1}^n Qj \right) = \chi(n+1)'Q(n+1)'. \quad (33)$$

The pivot array has 1 for the first element, and 0 for all of the other elements.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ -(\chi_1 + \chi_1') & -\chi_1' & -\chi_1' & -\chi_1' + \chi_4 & -(\chi_1' + \chi_4') \\ \chi_1 & -(\chi_2 + \chi_2') & -\chi_2' & -\chi_2' & -\chi_2' \\ 0 & \chi_2 & -(\chi_3 + \chi_3') & -\chi_3' & -\chi_3' \\ 0 & 0 & \chi_3 & -(\chi_4 + \chi_4') & -\chi_4' \end{bmatrix} \begin{bmatrix} Q1 \\ Q2 \\ Q3 \\ Q4 \\ Q5' \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (34)$$

III. CASE STUDIES

One common method of gas distribution is the use of a single-opening manifold with a number of evenly spaced holes of identical dimensions. In this case, all of the hole conductances are the same as are all of the conductances of the tube segments between them, ($Cn' = C'$, and $Cn = C$, therefore $\chi n' = \chi'$ and $\chi n = \chi$). The matrix therefore simplifies to

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \chi & -(\chi + \chi') & -\chi' & -\chi' \\ 0 & \chi & -(\chi + \chi') & -\chi' \\ 0 & 0 & \chi & -(\chi + \chi') \end{bmatrix} \begin{bmatrix} Q1 \\ Q2 \\ Q3 \\ Q4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (35)$$

Because the system is a chain of many repeated units (each unit can be thought of as forming an "L" composed of a

grounded element downstream from a series element), there is a high degree of symmetry. From a mathematical point of view, the solution is rather simple, but algebraically cumbersome. That the result can be expressed as a summation based upon binomial coefficients, and is summarized by Eq. (36). The formula has been proven for systems with as many as 24 holes. The expression determines what fraction of the total injected flow will flow through the i th hole of a manifold with n holes all of the same flow impedance, and whose tube conductances are also the same:

$$f(i, n) = \frac{\chi^{(i-1)} \sum_{j=0}^{n-i} \frac{(j+n-i)!}{(n-i-j)!(2j)!} \chi'^j \chi^{(n-i-j)}}{\sum_{j=1}^n \frac{(j+n-1)!}{(n-j)!(2j-1)!} \chi^{(n-j)} \chi'^{(j-1)}}. \quad (36)$$

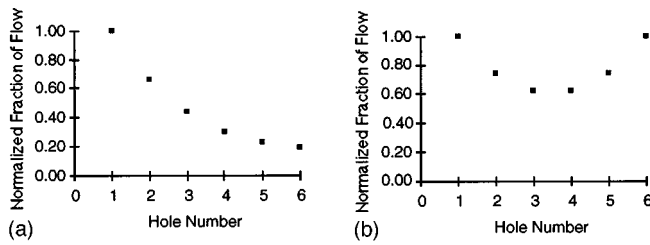


Fig. 3. (a) Normalized flow fraction through single-opening manifold. (b) Normalized flow fraction through double-opening manifold. The manifold dimensions are six holes spaced 2.3 in. apart, 0.1875 in. tube diameter, and 0.030 in. hole diameter.

Equation (36) is quite useful in calculating the gas flow distribution, by eliminating the work required to solve large matrices.

The following numerical example is presented to illustrate the difference between the two topologies; it assumes $T=300$ K, and N_2 gas. Six holes of 0.76 mm diameter spaced 5.84 cm apart are drilled into a 1/4 in. o.d. (4.7 mm i.d.) stainless-steel tube to form a single-opening manifold for gas injection into a reactor. The conductance of the 4.7 mm i.d. is 0.440 ℓ/s , $\chi=10.5$ s/ ℓ ; the spray hole conductance is 0.0178 ℓ/s , $\chi'=56.18$ s/ ℓ . Solving for these conductances produces the following result seen in Fig. 3(a), and the values are

TABLE I. Gas distribution through a six-hole single-opening manifold (spray hole conductance 0.018 ℓ/s , tube conductance 0.44 ℓ/s).

Hole No.	Single-opening manifold		Double-opening manifold	
	Fraction of flow	Normalized fraction of flow	Fraction of flow	Normalized fraction of flow
1	0.3544	1.0000	0.2110	1.0000
2	0.2334	0.6585	0.1569	0.7433
3	0.1561	0.4404	0.1321	0.6259
4	0.1081	0.3050	0.1321	0.6259
5	0.0803	0.2267	0.1569	0.7433
6	0.0677	0.1909	0.2110	1.0000

listed in the first half of Table I. The plot shows that five times as much gas flows through the first hole as flows through the last hole.

The other example is that of a double-opening manifold with the same dimensions as the six-hole example previously discussed. In this case, with six holes of 0.76 mm diameter spaced 5.84 cm apart are drilled into a 1/4 in. o.d. stainless-steel tube for gas admission into the reactor. In this case all hole conductances are the same, as are all of the conductances of the tube segments between them ($Cn'=C'$, and $Cn=C$, $\therefore \chi n'=\chi'$ and $\chi n=\chi$). The matrix simplifies to

$$\begin{bmatrix}
 1 & 1 & 1 & 1 & 1 & 1 & 0 \\
 -(\chi+\chi') & -\chi' & -\chi' & -\chi' & -\chi' & -\chi'+\chi & -2\chi' \\
 \chi & -(\chi+\chi') & -\chi' & -\chi' & -\chi' & -\chi' & -\chi' \\
 0 & \chi & -(\chi+\chi') & -\chi' & -\chi' & -\chi' & -\chi' \\
 0 & 0 & \chi & -(\chi+\chi') & -\chi' & -\chi' & -\chi' \\
 0 & 0 & 0 & \chi & -(\chi+\chi') & -\chi' & -\chi' \\
 0 & 0 & 0 & 0 & \chi & -(\chi+\chi') & -\chi'
 \end{bmatrix}
 \begin{bmatrix}
 Q1 \\
 Q2 \\
 Q3 \\
 Q4 \\
 Q5 \\
 Q6 \\
 Q7'
 \end{bmatrix}
 =
 \begin{bmatrix}
 1 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}. \quad (37)$$

As of now, no summation formula such as Eq. (36) has been derived for the double-opening manifold, so Gaussian elimination is employed. The symmetry is similar to the previous case, with the exception of the connecting element. The plot of the data is shown in Fig. 3(b), and the tabulated data are listed in the last two columns of Table I. Notice that the variation in the distribution of flow in the tube is only 40%, as opposed to 80% for the single-opening manifold. It is possible to compare the six-hole single-opening and the six-hole double-opening manifold by imagining that the single-opening tube manifold is shaped as a ring. The difference between the tube and double-opening manifolds would only be that the single-opening manifold has a wall blocking one end adjacent to the gas feed point; it would be absent for the double-opening manifold; see Fig. 4.

There is a way to conceptualize the reasoning behind the relative uniformity of the two topologies. For the single-

opening manifold, gas flows through the line with a conductance given by the cross section of the tube. However, for the double-opening manifold, gas flows in two directions at once so that the cross section available for gas flow at the junction is *effectively* doubled. There is no change in tube cross section, but the additional direction of gas flow provides an extra path for the gas to be transported to each hole. It can be thought of as adding an extra flow bar at the junction and shortening the transport length of gas to any exhaust hole.

One common misconception that arises from this line of reasoning, however, is that, since the gas flow is evenly divided and if there is a hole exactly half way along the tube from the injection point, there will be no gas output from that hole since there will be no pressure gradient in the tube. It is correct that there is no tube pressure gradient, but there still is a pressure gradient between the tube and the reactor that causes gas to flow into the reactor at the halfway point on the ring.

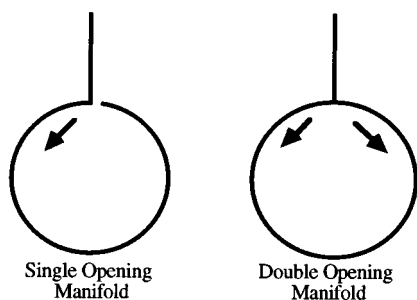


FIG. 4. Similarities in the shapes of the two manifold topologies. The arrows indicate directions of gas flow within the manifold tube. Note that the tube manifold only connects with the feed tube with only one feed path, while the double-opening manifold connects with the feed tube with two feed paths.

IV. MANIFOLD OPTIMIZATION

A. Uniform output

Quite often, it is desirable that the gas flow be evenly distributed through the holes of the manifold, i.e., $Q_i = Q/n$. This is a reasonable goal, for example, in cases in which the holes in the ring are on a plane that is normal to the bulk gas stream. As shown above it is easy to choose arbitrary dimensions that produce a manifold with nonuniform output. One inference of the data is that manifolds with uniform hole size and spacing cannot have uniform gas distribution if it is a single-opening manifold with more than one hole, or an double-opening manifold with more than two holes. This section will discuss ways of producing manifolds that distribute the gas evenly across the manifold.

Figure 5 compares the maximum difference in flow between 12-hole single- and double-opening manifolds for a wide range of conductance ratios. Though the general behavior of the plots is the same, the double-opening manifold maintains a more uniform flow for each given conductance ratio. In addition, the plot shows that, to achieve better uniformity in both cases, the tube to hole conductance ratio should be maximized. By increasing the conductance ratio, the pressure drop between segments of the tube is minimized relative to the pressure drop across the hole. Minimizing the pressure drops minimizes the pressure drop variation from hole to hole. The simplest ways to do this are to decrease the

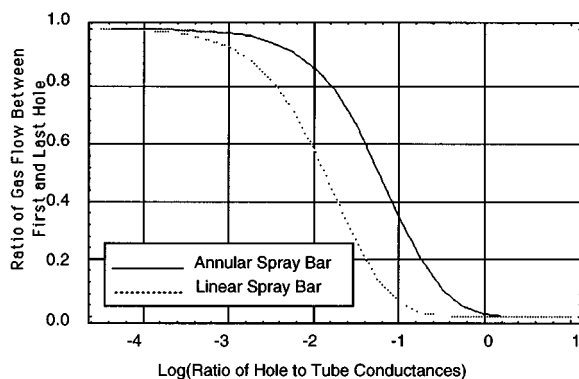


FIG. 5. Comparison of the single- and double-opening manifolds minimum/maximum flow distribution ratio for hole to tube conductance ratios of $\sim 1:30\,000$ to $1:0.1$, for 12 holes.

hole diameter, increase the hole length, and increase the inner diameter of the feed tube. This shows that double-opening manifolds produce more even flow distributions than single-opening manifolds.

The number of holes employed in a manifold determines how uniform the gas flow is distributed. It can be shown that the greater number of holes that are used for a manifold, the larger the ratio of the flow between the first and last hole in the manifold. For a given length of tubing, the greater number of holes that exist, the greater the hole density along the tube. Since each tube is an opportunity for gas removal, the greater the number of holes, the more depletion that occurs within a given segment of tube. Conversely, the fewer holes used, the more uniform the *distribution of gas flow* through each hole. Of course even though the gas distribution through each hole may be similar, it says nothing about the distribution of the gas in the reactor. The lower the diffusion coefficient of the diluent, the more susceptible it will be to a nonuniform concentration profile within the reactor. Therefore, a set of tolerances of diluent concentration must be defined and the diffusion dynamics must be known in order to determine what the maximum desirable spacing of holes can be. This balance is dependent upon the plug velocity of the reactor gas stream, the diffusion coefficient of the injected gas in the stream, and the transport distance of interest.

B. Nonuniform output

In theory it is possible to design a manifold that produces any desired flow distribution based upon the tube and hole conductances. In reality though, certain designs would be impractical to fabricate. One approach to tailoring gas flow distribution would be to use a large conductance tube to minimize the pressure drop across the manifold, so that the flow through each hole would approach that of the ratio of the particular hole conductance over the sum of all the hole conductances.

$$f(i,n) \approx \frac{C_i}{\sum_{j=1}^n C_j} \quad C' \gg C_i(\max). \quad (38)$$

If C' has to be within the same range as $C_i(\max)$, it becomes difficult to design the manifold so that it has the desired flow distribution, because the tube conductances must be included in the calculations. In such a case, the only solution is to solve the appropriate matrix given the desired flows.

V. SUMMARY

The methodology for determining the flow distribution through a gas manifold has been presented. Two topologies have been examined, single- and double-opening manifolds in which there is only one gas feed point. It was shown that the double-opening manifold tended to provide more uniform flow distribution for similar conductance ratios and number of holes, compared to the single-opening manifold. A formula was presented that calculates the gas distribution values for a single-opening manifold in which the holes' conductances are all the same, and the tube conductances be-

tween the holes are all the same. There are three rules to consider when designing a manifold that minimizes nonuniformity in the gas flow distribution.

- (1) Use tube dimensions such that the tube/spray hole conductance ratio is maximized.
- (2) Use as few holes in the manifold as possible.
- (3) Use a double-opening manifold when possible.

It is possible to tailor the gas flow distribution by varying the conductances of the various tube segments and spray holes by solving the appropriate matrix. The most practical solution, however, would be to use a tube with a large conductance relative to the hole conductances and make the hole conductances proportional to the fraction of gas flow through them.

This article has demonstrated how network theory can be used to solve gas flow distribution problems through flow tubes, and to determine generalized criteria for designing

tubes with uniform gas flow distribution. This is a valuable first step in a generalized theory of manifolds, and it provides a basis from which to develop a generalized network for analyzing reactor flow modeling.

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